

Bifurcation analysis of DDEs: an introduction to DDE-BIFTOOL by examples

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Udine, 6 aprile 2018

DDE-BIFTOOL: information

home page: <http://ddebiftool.sourceforge.net/>

current version: 3.1.1

main contributors:

- original code and documentation:
K. Engelborghs, T. Luzyanina, G. Samaey, D. Roose,
K. Verheyden (K.U. Leuven)
- revision, bifurcations of periodic orbits, current maintainer:
J. Sieber (U. Exeter)
- normal form coefficients for bifurcations of equilibria:
S. Janssens, B. Wage, M. Bosschaert, Yu.A. Kuznetsov
(U. Utrecht)

DDE-BIFTOOL: capabilities

bifurcation analysis of delay differential equations
with finite number of constant or state-dependent discrete delays

- 1-par continuation of equilibria
- characteristic roots
- 2-par continuation of folds and Hopf bifurcations
- 1-par continuation of periodic solutions
- Floquet multipliers
- 1-par continuation of secondary branches of periodic solutions (at folds or period doubling bifurcations)
- 2-par continuation of folds and period doubling bifurcations
- computation of normal form coefficients
- computation of homoclinic and heteroclinic orbits (only constant delays)

DDE-BIFTOOL: structure

data structures:

- system definition
- points
- stability
- method parameters
- branches
- scalar measures

low-level functions:

- point manipulation
- branch manipulation
- normal forms

high-level functions:

- branch initialization
- normal forms
- other common operations

DDE-BIFTOOL: resources

install: download from <https://sourceforge.net/projects/ddebiftool/>
and unzip

learn:

- manuals
- demos
- tutorials by Maikel Bosschaert

[https://www.uhasselt.be/UH/Computational-Mathematics/
Members-of-the-research-group/Maikel-Bosschaert.html](https://www.uhasselt.be/UH/Computational-Mathematics/Members-of-the-research-group/Maikel-Bosschaert.html)

material for this seminar:

download from <https://users.dimi.uniud.it/~davide.liessi/>
and unzip

examples

- a population model by Cooke, van den Driessche and Zou

J. Math. Biol. 39 (1999), pp. 332–352, DOI:10.1007/s002850050194

$$x'(t) = b e^{-ax(t-\tau)} x(t-\tau) e^{-d_1 \tau} - dx(t)$$

- the Mackey–Glass equation

Science 197:4300 (1977), pp. 287–289, DOI:10.1126/science.267326

$$x'(t) = \beta \frac{x(t-\tau)}{1 - x(t-\tau)^n} - \gamma x(t)$$

- a surprisingly difficult equation...

$$x'(t) = ax(t) + bx(t-\tau)$$